Taylor Polynomials and Power Series Calculus of a Single Variable X. Du

- **Taylor Polynomials**
    - For a function $f(x)$ with n derivatives at c, the n^{th} degree Taylor polynomial centered about $x = c$ is

      $$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \ldots + \frac{f^n(c)}{n!}(x-c)^n$$
    - Or equivalently: $P_n(x) = \sum_{i=0}^{n} \frac{f^i(c)}{i!}(x-c)^i$
    - $f(x)$ must be differentiable n times on its domain
    - **Maclaurin Polynomials** are Taylor Polynomials centered about $x = 0$

      $$P_n(x) = \sum_{i=0}^{n} \frac{f^i(0)}{i!} x^i$$
    - Taylor polynomials are used to approximate a function.
    - Linear approximation uses a first-degree Taylor polynomial.
    - The equation of the tangent line at (c, $f(c)$) is the first-degree Taylor polynomial centered about $x = c$.
- **Taylor's Theorem**
    - If a function $f(x)$ is differentiable through order $n+1$ in an interval I containing c, then for each $x \in I$, there exists a z between x and c such that

      $$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \ldots + \frac{f^n(c)}{n!}(x-c)^n + R_n(x) \text{ where}$$

      $$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-c)^{n+1}.$$
    - Relate to Mean Value Theorem, except with multiple derivatives!
    - $|R_n(x)|$ is the error of your approximation
- **Power Series**
    - Expresses a polynomial as a series
    - Infinite-degree Taylor polynomial (Taylor series)
    - $f(x) = \sum_{n=0}^{\infty} \frac{f^n(c)}{n!}(x-c)^n$ for a power series centered about c
    - **Radius of Convergence**
        - R is the value such that the series converges for all $|x-c| < R$ and diverges for all $|x-c| > R$
        - To find R, use ratio test! Compute $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$
    - **Interval of Convergence**
        - Ranges from $c - R$ to $c + R$
        - Check endpoints $x = c \pm R$ to determine inclusive or exclusive
    - Use power rule normally to find derivatives and integrals.